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Performance of Some Procedures for Detection of Many Outliers in Samples

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Abstract: For detecting multiple outliers in normal samples we consider some tests and their performance and also depicted that which procedure is more powerful in the considered situation. The considered test statistics ***are ESD test, Grubb's test(*** E_k, L_K and L_k^****) and Rosner R-Statistics(RST)***. Here the power of these test statistics are studied and necessary discussion are made on the basis of computed result. Although ***ESD*** is an one outlier procedure we have used here to study the performance of many outlier procedure in case of detecting single outlier.

Key words: Multiple outliers, ESD test, Grubb's test, RST, simulation, power.

1. INTRODUCTION

The problem of testing outlying observations, is of considerable importance in applied statistics. Many and various types of significance tests have been proposed by Statistician interested in the field of application. 1933, Rider published a rather comprehensive survey of work on the problem of testing the significance of outlying observations up to that date. The test criteria surveyed by Rider appear to impose as an initial condition that the standard deviation, σ, of the population from which the items were drawn should be known accurately.

Irwin's criteria (1925) which utilize the difference between the first two individuals or the difference between the second and third individuals in random samples from a normal population. The range or maximum dispersion of a sample which has been advocated by Student (1927) and others for testing the significance of outlying observation . Mackey (1935) published a note on the distribution of the last mention statistics and by means of a rather elaborate procedure obtained a recurrence relation between the distribution of the extreme minus the mean in a sample of n from a normal universe and the distribution of this statistics in sample of n-1 from the parent. Mckey gave also an approximate expression for the upper percentage points of the distribution but did not tabulated the exact distribution due to the complexity of the multiple integrals involved. Nair (1948) has tabulated the distribution of the difference between the extreme and sample mean for n = 2 to n = 9.

Under certain circumstances, accurate knowledge concerning σ may be available as, for example, in using " daily control" tests the population standard deviation may be estimated in some cases with sufficient precision from past data. In general, however, as accurate estimate of σ may not be available and it becomes necessary to estimate the population standard deviation for the single sample involved or "studentized" the statistics to be need, thus providing a true measure of the risks involved in the significance test advocated for testing outlying observation . Thompson (1935) apparently had this vary point in mind when he devised an exact test in his paper "On a criteria for the rejection of observation and the distribution of the ratio of the deviation to the sample standard deviation," which appeared in 1935

Pearson and Chandra Sekar (1936) have given a rather comprehensive study of Thompson's criteria. They discussed also some very important view points which should be taken into consideration when dealing with the problem of testing outlying observations. Pearson and Chandra Sekar point out that if only one of the observation actually came from a population with divergent mean, then Thompson criteria would be very useful, whereas if two or more of the observations are truly outlying then the criteria may be quite ineffective, particularly if the sample contain less than about 30 or 40 observations.

Once the sample results of an experiment are available, the practicing statistician may be confronted with one or more of the following distinct situations as regard discordant observations

(a) To begin with, a very frequent or perhaps situations that either the greatest observation or the least observation in a sample may have the appearance of belonging to a different population then the one from which the remaining observations were drawn. Here we are contracted with tests for a single outlying observation.
(b) Then again, both eth largest and the smallest observations may appear to be 'different 'from the remaining items in the sample. Here we are interested in testing the hypothesis that both the largest and the smallest observations are truly outliers
(c) Another frequent situation is that either the two largest or the two smallest observations may have the appearance of being discordant. Here we are interested in reaching a decision as to whether we should reject the two largest or the two smallest observations as not being representative of the ting we are sampling.

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As to why the discordant observations in a sample may be outlier, this may be due to error of measurement in which case we would naturally want to reject or at least "correct" such observations. On the other hand, it may be that the population we are sampling is not homogeneous in the uni-modal sense and it will consequently be desirable to know this so that we may carry out further development work on our product if possible or desirable.

Grubbs (1950) made a historical comment regarding the outliers and formulate the problem to arrive at a situation. He developed two statistics for testing the significance of the largest observation and the smallest observation in a sample of size n from a normal population.

Some Grubbs-Type statistics for the detection of several outliers were developed by Tietjen and Moore (1972). They also gave the table of critical values to facilitated the use of these statistics. Rosner (1975) gave a flexible procedure which can detect from 1 up to k outliers and is useful for a specific number of outliers which is true of the procedures. He finds that his approach yields a more powerful procedure than the one outlier procedures extreme studentized deviate (ESD). ESD procedure is essentially a stepwise Grubbs (1950) test for a single outlier in either tailed is a powerful test procedure for the significance in the sample.

In this paper we have studied the power of statistics viz Tietjen and Moore (E_k), Rosner (RST), Grubbs, ESD, L_k and L_k^* and necessary discussion are made on the basis of computed results. Although ESD is an one outlier procedure we have used here to study the performance of many outlier procedure in case of detecting single outlier .

2.TEST PROCEDURES

Let $X_1, X_2, \ldots, X_N$ be a sample drawn from the normal distribution. Suppose there are k outliers. These may be all greater than mean or smaller than mean or out of k some are smaller than mean or some are larger than mean. Let n-k observations follow Nor(μ_0,σ^2), and k observations follow Nor(μ_1,σ^2) if less than mean and Nor(μ_2,σ^2) if greater than mean and follow both Nor(μ_1,σ^2) and Nor(μ_2,σ^2) if les in both side of the mean . Here parameters are unknown. From these observations, we are to detect k smallest or k largest or out of k some are smallest and some are largest observations which we consider as outliers. Now the observations are arranged according to ascending order and denote these as,

$$y_1 \le y_2 \le \ldots \le y_N$$

2.1. Extreme Studentized Deviate (ESD) Test:

The ESD procedure is essentially a stepwise Grubbs T- test for a single outlier in either tail. At each step, the ESD is calculated

$$ESD_i = \max_{j=1,2,\ldots,N-i+1} \left| y_i - \bar{y}_i \right| \Big/ s_i \qquad , i = 1,2,\ldots,k$$

Where $\bar{y}_i$ and s_i^2 are the mean and variance, respectively, of the sub-sample remaining after the first i-1 step and observation deletions. If the value of ESD is less than the tabulated value at the desired level of significance, then declare no outliers detected; if greater than tabulated value then declare y_N as an outlier, delete y_N from the sample and perform the ESD test on the reduced sample of size (N-1) and declare y_{N-1} an outlier if and only if the latter ESD test is significant. A downfall of this method is that it require the assumption of a normal data distribution. Usually this assumption holds true as the sample size gets larger.

2.2. L_k, L_k* and E_k Test :

Tietjen and Moore (1972) generalize the Grubbs' two outliers procedure to detecting k outliers in the sample. The null hypothesis that all the values have come from the same normal distribution against the alternative hypothesis that there are k larger observations have come normal distribution with different mean. That means these are outliers which are different from the remaining observations. The proposed statistics is

$$L_k = \frac{\sum_{i=1}^{N-k} (y_i - \bar{y}_k)^2}{\sum_{i=1}^{N} (y_i - \bar{y})^2}$$

where $\bar{y}_k = \dfrac{1}{N-k} \sum_{i=1}^{N-k} y_i$ and

$$\bar{y} = \frac{1}{N} \sum_{i=1}^{N} y_i$$

This L_k statistics can be used for examining the k largest observations . The percentage points of this statistic are available in Tietjen and More(1972).

To used the statistic for k smallest observations as outliers above statistic L_k can be used by modifying the numerator by including the (N-k) largest values. If we denote this statistic as L_k* ,it can written as

$$L_k^* = \frac{\sum_{i=k+1}^{N} \left(y_i - \bar{y}_k^*\right)^2}{\sum_{i=1}^{N} (y_i - \bar{y})^2}$$

where

$$\sum_{i=k+1}^{N} \left(y_i - \bar{y}_k^*\right)^2 , \qquad \text{where}$$

$$\bar{y}_k^* = \frac{1}{N-k} \sum_{i=k+1}^{N} y_i$$

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So, L_k and L_k^* statistics are used for testing k suspected values which are larger or smaller than the bulk of the sample. There is also another situation in which out of the k outliers some suspected values are smaller and some are larger than the remaining values. In this situation, the statistics E_k is preferred. To obtain the statistics E_k, let us take the sample values $x_1, x_2, \ldots \ldots x_N$. Then compute the mean of the sample, $\bar{x}$. Then obtain the absolute residuals like-

$r_1 = |x_1 - \bar{x}|, \quad r_2 = |x_2 - \bar{x}|, \ldots\ldots\ldots r_n = |x_N - \bar{x}|$.

Now denote these observations as z's in such a manner that z_i is the x whose r_i is the i^{th} largest.

Here, z_1 indicates the observation which closest to the mean i.e. which difference is small, z_N indicates the observation which farthest from the mean i.e. which difference is large.

The proposed statistics for testing the null hypothesis that all the values come from the same normal distribution is

$$E_k = \frac{\sum_{i=1}^{N-1}\left(z_{(i)} - \bar{z}_k\right)^2}{\sum_{i=1}^{N}\left(z_i - \bar{z}\right)^2} \quad where \quad \bar{z}_k = \frac{1}{N-k}\sum_{i=1}^{N-k}\left(z_{(i)}\right)$$

$$\text{and} \quad \bar{z} = \frac{1}{N}\sum_{i=1}^{N} z_i$$

i.e. $\bar{z}_k$ and $\bar{z}$ indicates the mean of the (N-k) least extreme observations and the mean of all observations respectively. Table value of L_k* and E_k statistics are also available in Tietjen and Moore(1972).

2.3. Rosner R-Statistic (RST):

Several procedure exist for the detection of a specific number of outliers. However, the more typical problem is that one does not know the number of outliers in advance and one wants a procedure which is reasonably powerful in detecting varying numbers of outliers. Furthermore, procedures that are designed for a specific number of outliers often have little power if a different number of outliers is present. Rosner (1975) proposed a flexible procedure which can detect from 1 up to k outliers . Suppose we are interested in a procedure which can detect at most k outliers where k = [pN] for some fraction of p of the total sample size of N. To implement this procedure, delete the k smallest and k largest observations from the sample and calculate the mean 'a' and standard deviation 'b' of the trimmed sample. Then calculate the largest studentized residual in absolute value (R_1) of the entire sample, using 'a' and 'b' instead of $\bar{X}$ and s. Define R_2 to be the second largest residual etc.

Specifically, let

$$a = \sum_{i=k+1}^{N-k} \frac{X_{(i)}}{N-2k} = \text{trimmed mean}$$

$$b^2 = \sum_{i=k+1}^{N-k} \frac{(X_{(i)} - a)^2}{(N-2k-1)} = \text{trimmed variance}$$

Where $X_{(i)}$ is the ith order statistics in a sample of size N

Let

$I_0 = \{X_1, X_2, \ldots\ldots\ldots X_N\}$,

$$R_1 = \max_{i \in I_0} \frac{|X_i - a|}{b} = \frac{|X^{(0)} - a|}{b}$$

$I_1 = I_0 - X^{(0)}$, $\quad R_1 = \max_{i \in I_1} \frac{|X_i - a|}{b}$,……..

$$R_k = \max_{i \in I_{k-1}} \frac{|X_i - a|}{b} = \frac{|X^{(k-1)} - a|}{b}$$

Where $I_q = I_0 - \{X^{(0)}, X^{(1)}, \ldots\ldots X^{(q-1)}\}$, q = 2,………..k-1

In this procedure, consider the marginal distributions of $R_1, \ldots\ldots\ldots\ldots, R_k$ and specifying find β, $\lambda_1(\beta), \ldots\ldots\ldots\ldots \lambda_k(\beta)$ such that

$\Pr[R_i > \lambda_i(\beta)] = \beta, \quad i = 1, 2, \ldots \ldots \ldots k$ and

$$\left\{\bigcup_{i=1}^{k}\left[R_i \geq \lambda_i(\beta)\right]\right\} = \alpha \text{, Where } \alpha \text{ is}$$

the desired significance level.

The RST many outlier procedure then has the following form: if the event $\left\{\bigcap_{i=1}^{k}\left[R_i \leq \lambda_i(\beta)\right]\right\}$ is true, then we declare that no outliers are present; if there is at least one i such that $R_i > \lambda_i(\beta)$, i =1, 2,………k, i.e., if $\left\{\bigcup_{i=1}^{k}\left[R_i > \lambda_i(\beta)\right]\right\}$ is true and $l = \max\{i : R_i > \lambda_i(\beta)\}$ then we declare $X^{(0)}, X^{(1)}, \ldots\ldots\ldots, X^{(i-1)}$ are outliers.

The motivation behind this procedure is that one computes measures of location (a) and scale (b) from the set of points that cannot possibly be outliers, i.e. the points that remain after deleting 100*p*% of the sample from each end. One then declares one of the statistics $R_1, R_2, \ldots\ldots R_k$ is large relative to its distribution. This chosen RST procedure is computationally the least cumbersome, and is about equivalent in power for a variety of alternatives to any of the many outlier procedures.

Monte Carlo Study:

To study the power of test statistics we generate a random sample from standard normal distribution N(0,1) using Box and Muller(1958) technique then adding μ_1, μ_2 in two observations make these as outliers and μ_1 to one observation to make one outlier respectively.. For each set of observations, value of the statistics are calculated and compare with the theoretical value to accept or reject the hypothesis. If it is rejected it is counted and repeat the process. We have repeated 10,000 time for each sample size and calculated the proportion of rejected

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i.e number of rejected the null hypothesis divided by the total number of repetition is calculated and tabulated in different tables.

Table 1. Empirical power of ESD, E_k and RST for n =10

(μ_1, μ_2)	**level**	**ESD**	**E_k**	**RST**
(0,0)	.01 .05	.0123 .0618	.0087 .0509	R_1=.0065 R_2=.0069 R_1=.0337 R_2=.0396
(0,4)	.01 .05	.48 .6624	.0999 .3126	R_1=.0424 R_2=.1825 R_1=.0667 R_2=.2529
(0,6)	.01 .05	.9025 .9710	.4532 .8115	R_1=.1607 R_2=.2758 R_1=.4767 R_2=.6776
(4,4)	.01 .05	.052 .1560	.0087 .0509	R_1=.0000 R_2=.0000 R_1=.0000 R_2=.0001
(4,6)	.01 .05	.2226 .4459	.0125 .0681	R_1=.0000 R_2=.0000 R_1=.0003 R_2=.0006
(-4,4)	.01 .05	.1577 .3982	.8268 .9851	R_1=.0217 R_2=.0512 R_1=.1110 R_2=.2599
(-4,6)	.01 .05	.4038 .697	.9729 .9995	R_1=.059 R_2=.142 R_1=.2661 R_2=.5175
(-5,5)	.01 .05	.1544 .454	.9727 .9995	R_1=.0418 R_2=.1158 R_1=.2054 R_2=.4546

Table 2. Empirical power of ESD, E_k and RST for n = 20

(μ_1,μ_2)	**Level**	**ESD**	**E_k**	**RST**
(0,0)	.01 .05	.0141 .0561	.0084 .046	R_1=.0055 R_2=.0048 R_1=.0279 R_2=.0380
(0,4)	.01 .05	.5932 .7695	.3473 .5948	R_1=.2292 R_2=.2905 R_1=.5273 R_2=.5424
(0,6)	.01 .05	.9697 .9921	.9285 .9859	R_1=.7648 R_2=.8702 R_1=.9510 R_2=.9684
(4,4)	.01 .05	.3215 .6501	.0084 .046	R_1=.0007 R_2=.0032 R_1=.0127 R_2=.0328
(4,6)	.01 .05	.7373 .9374	.0264 .0971	R_1=.0097 R_2=.0399 R_1=.0853 R_2=.1951
(-4,4)	.01 .05	.507 .8221	.9997 1.000	R_1=.1813 R_2=.3969 R_1=.5025 R_2=.7434
(-4,6)	.01 .05	.8626 .9827	1.000 1.000	R_1=.574 R_2=.8151 R_1=.8887 R_2=.9683
(-5,5)	.01 .05	.7245 .9581	1.000 1.000	R_1=.4256 R_2=.7509 R_1=.8016 R_2=.9520

Table 3. Empirical power of ESD, E_k and RST for n = 30

μ_1,μ_2)	level	ESD	E_k	RST
(0,0)	.01 .05	.0117 .0586	.0105 .0497	R_1=.0066 R_2=.0048 R_1=.0299 R_2=.0269
(0,4)	.01 .05	.6437 .7803	.4755 .6944	R_1=.3649 R_2=.3817 R_1=.6469 R_2=.5957
(0,6)	.01 .05	.9844 .9945	.9831 .9974	R_1=.9248 R_2=.9512 R_1=.9900 R_2=.9865
(4,4)	.01 .05	.5709 .8017	.0105 .0497	R_1=.0184 R_2=.0433 R_1=.1196 R_2=.2113
(4,6)	.01 .05	.9200 .9848	.0359 .1174	R_1=.1403 R_2=.2960 R_1=.4572 R_2=.6247
(-4,4)	.01 .05	.6972 .8901	1.000 1.000	R_1=.3486 R_2=.6021 R_1=.6995 R_2=.8525

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(-4,6)	.01 .05	.9608 .9944	1.000 1.000	R_1=.8463 R_2=.9483 R_1=.9779 R_2=.9925
(-5,5)	.01 .05	.9222 .9855	1.000 1.000	R_1=.7155 R_2=.9269 R_1=.9409 R_2=.9895

Table 4. Empirical power of ESD and L_k for n = 10

(μ_1,μ_2)	Level	ESD	L_k
(0,0)	.01 .05	.0123 .0518	.0101 .0529
(2,2)	.01 .05	.0463 .1234	.0306 .1359
(3,3)	.01 .05	.057 .1494	.1082 .3364
(4,4)	.01 .05	.052 .156	.2818 .6363
(5,5)	.01 .05	.038 .1444	.5337 .861
(6,6)	.01 .05	.0249 .1235	.7583 .964
(7,7)	.01 .05	.013 .1012	.9016 .9943

Table 5. Empirical power of ESD and L_k for n = 20

(μ_1,μ_2)	Level	ESD	L_k
(0,0)	.01 .05	.0141 .0561	.0087 .0537
(2,2)	.01 .05	.0791 .207	.0597 .1978
(3,3)	.01 .05	.1833 .4062	.2528 .528
(4,4)	.01 .05	.3215 .6501	.6158 .8564
(5,5)	.01 .05	.4648 .8441	.8953 .9814
(6,6)	.01 .05	.6037 .956	.986 .9995
(7,7)	.01 .05	.7256 .9908	.9997 1.000

Table 6. Empirical power of ESD and L_k for n = 30

(μ_1,μ_2)	level	ESD	L_k
(0,0)	.01 .05	.0117 .0546	.0096 .0498
(2,2)	.01 .05	.1024 .2262	.0677 .1985
(3,3)	.01 .05	.2875 .4972	.3164 .5580
(4,4)	.01 .05	.5709 .8017	.7260 .8920
(5,5)	.01 .05	.8293 .9569	.9568 .9904
(6,6)	.01 .05	.9567 .9960	.9975 .9997
(7,7)	.01 .05	.9944 .9999	.9999 1.000

Table 7. Empirical power of ESD and L_k^* for n = 10

(μ_1,μ_2)	Level	ESD	L_k^*
(0,0)	.01 .05	.0123 .0518	.0105 .0494
(-2,-2)	.01 .05	.0500 .1216	.0332 .1390
(-3,-3)	.01 .05	.0572 .1443	.1165 .3436
(-4,-4)	.01 .05	.0496 .1547	.2961 .6260
(-5,-5)	.01 .05	.0360 .1458	.5347 .8529
(-6,-6)	.01 .05	.0219 .1241	.7583 .9601
(-7,-7)	.01 .05	.0126 .1003	.9039 .9935

Table 8. Empirical power of ESD and L^*_k for n = 20

(μ_1,μ_2)	Level	ESD	L_k^*
(0,0)	.01 .05	.0141 .0541	.0104 .0521
(-2,-2)	.01 .05	.0806 .2160	.0639 .1969
(-3,-3)	.01 .05	.1900 .4078	.2625 .5182
(-4,-4)	.01 .05	.3247 .6495	.6276 .8498
(-5,-5)	.01 .05	.4649 .8490	.8992 .9822
(-6,-6)	.01 .05	.6019 .9568	.9890 .9992
(-7,-7)	.01 .05	.7250 .9921	.9993 1.000

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Fig.1 Empirical Power of The Test Statistics ESD, E_k and RST for n=10

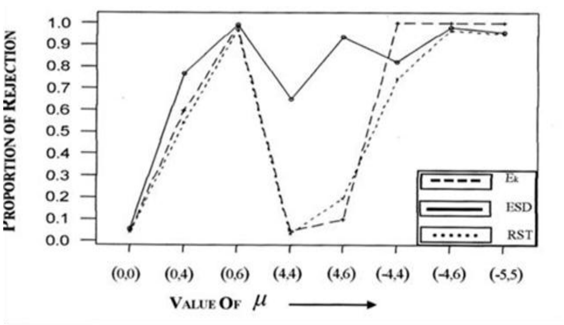


Fig. 2 Empirical Power of the Test Statistics ESD, E_k and RST for**n=20**

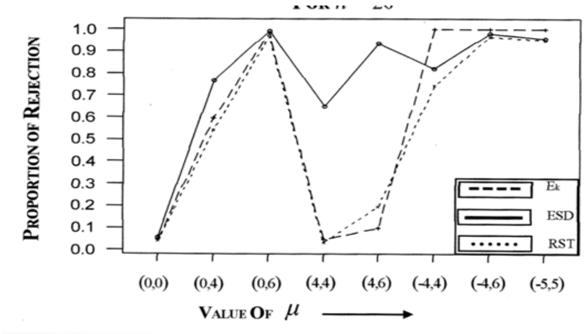


Fig.3 Empirical Power of The Test Statistics ESD, E_k and RST for n=30

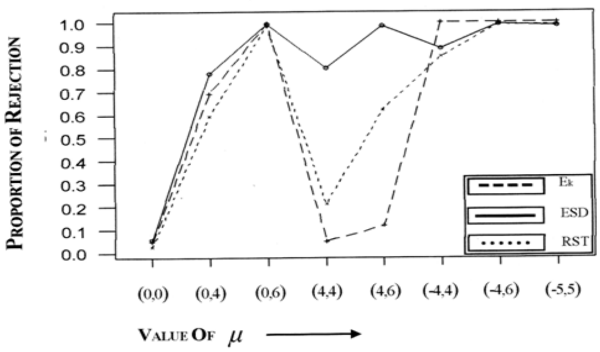


Fig 5. Empirical Power of the test Statistics ESD and L_k for n=20

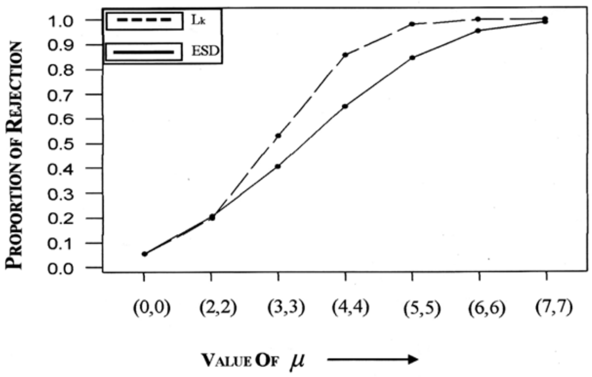


Fig.4 Empirical power of The Test Statistics ESD and L_k for n=10

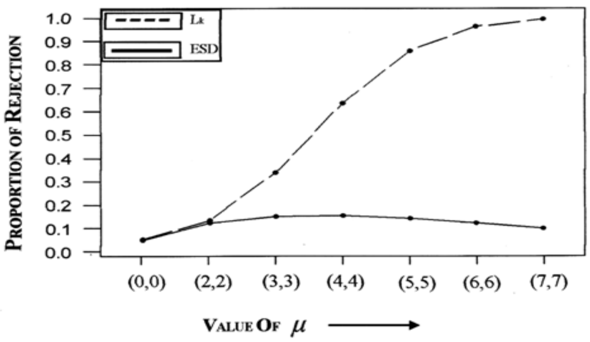


Fig.6 Empirical Power of the Test Statistics ESD and L_k for n=30

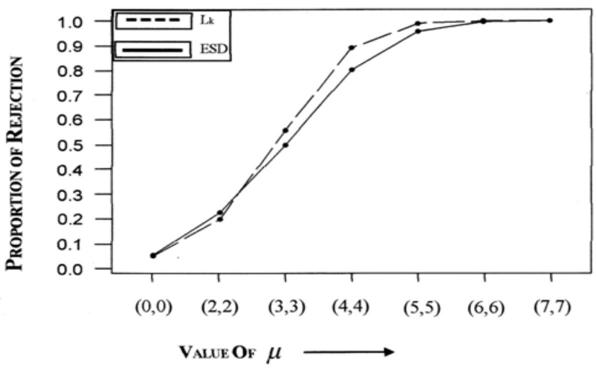


Fig.7 Empirical Power of The Test Statistics ESD and L_k^* for n=10

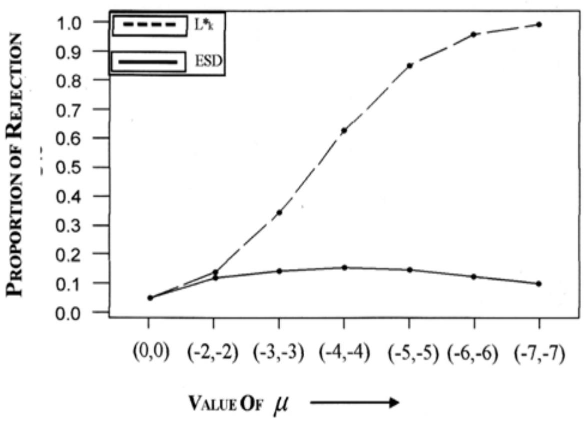
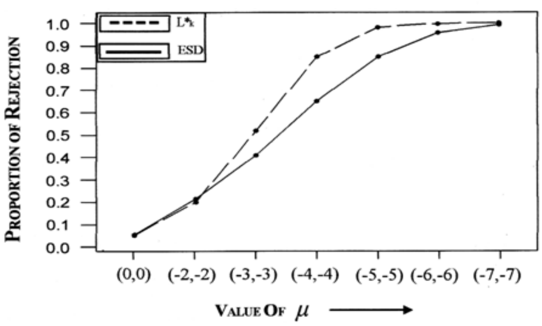


Fig.8 Empirical Power of The Test Statistics

Fig.8 Empirical Power of The Test Statistics ESD and L_k^* for n=20



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3.DISCUSSION

The ESD many outlier procedure seen to be slightly more powerful than their many outlier counterparts in case of alternative (0,4) and (0,6) i.e. for one outlier. On the other hand, the E_k procedure is more powerful when the outliers are in two sides of the mean. For example in table1, against the alternative (-5,5), the E_k procedure has overall power of .9995 while the ESD procedure has overall power of .454 at 5% level of significance. There is also one thing to noticeable that when the outliers are same side of the mean, i.e. (4,4) and (4,6) the power of RST procedure is negligible.

The same case arise in table 2 and table 3. Note that when the sample sizes increases, the power of all procedures increases simultaneously. For example in table 2 and table 3, the power of E_k procedure almost 1.0000 for alternative (-4,4), (-4,6) and (-5,5). That is, the probabilities of detecting many outliers for E_k procedure more than the other procedures.

In table 4, table 5 and table 6 we have computed power of ESD and L_k procedure for sample sizes N = 10,20 and 30 respectively. Here, we have seen that the power of L_k procedure is higher than the ESD procedure. It is observed that when the sample size increases the power L_k procedure also increases. For example in table 5 and table 6, for alternative (7,7), the power of L_k is 1.000. The same thing arise in L_k^* procedure that is shown in table 7 and table 8. Here the L_k^* procedure seems to be more powerful than the ESD procedure .As the sample size increases the corresponding power of L_k* and ESD also increases. Note that the L_k^* procedure is applicable when the outliers are on the left side of the mean. Similar case happen in L_k procedure because it is also applicable only when the outliers are on the right side of the mean. For this reason, ESD procedure seems to be weaker than L_k and L_k^* procedure.

Figures 1, 2 and 3 represents the empirical power of the test statistics ESD, E_k and RST at 5% level when n = 10, 20 and 30.

Figures 4, 5 and 6 represents the empirical power of test statistics ESD and L_k at 5% level
for n = 10, 20 and 30.

Similarly, figure 7 and 8 shows the empirical power of test statistics ESD and L_k^* at 5% level for n = 10 and 20.

4.CONCLUSION

We have come to the conclusion that in case of single outlier ESD procedure seems to be powerful than the other procedures. If outliers are lies in right hand side mean L_k is better than the ESD and If outliers are lies in left hand side mean, L_k* is more powerful than the ESD.

Furthermore, among the several multiple outlier procedures, E_k is found to be the best .The RST procedure is slightly less powerful than both E_k and ESD.

Example: We here give an example of how to use the multiple outlier procedures. We first generate 20 observations from normal distribution N(0, 1) and adding 5 to last two observations to introduce outliers .We have used the above procedures i.e ESD, L_k, E_k, and RST to see whether these procedures can detect these two outliers or not.

Table 9: Generated observations from normal distribution

Observation Number	Generated Sample	Observation Number	Generated Sample
1	1.92958	11	1.97983
2	1.63060	12	1.13361
3	0.21555	13	0.80564
4	-0.77804	14	1.32789
5	0.65219	15	0.42908
6	-2.010552	16	1.46078
7	0.59968	17	-1.54222
8	0.82207	18	-0.71746
9	-0.29068	19	5.43100
10	0.59058	20	4.36602

ESD Procedure:

Now for generated sample, Mean, $\bar{x}(I_0) = 0.902$

Standard deviation, $s(I_0) = 1.749$

$$R_1 = |5.43100 - 0.902| / 1.749 = 2.589$$

$$R_2 = |4.36602 - 0.663| / 1.424 = 2.6004$$

Here R_1 > 2.447 and R_2 > 2.426 at 10% point for n =20 and n =19 respectively. So, it is concluded that both 5.43100 and 4.36602 are outliers.

L_k Procedure:

For generated sample the value of

L_2 = 22.028/58.093 = 0.3792,

which is smaller than the 1% critical value of 0.387, so that the test rejects both 5.43100 and 4.36602 as outlier.

E_k Procedure:

Here the value of E_2 = 6.5582/18.155 = 0.36123, which is considerably less than the critical 5% value of 0.416, so that E_2 would reject the two observations simultaneously.

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RST Procedure:

To apply this procedure, first choose the following observations as outliers and then calculate : -2.01055, -1.54222, 4.36602 and 5.43100

$$a = \frac{-0.77804 - 0.71746 + ...+ 1.97983}{16}$$

$= 0.773$

b = 0.8086

$$R_1 = |5.43100 - 0.773| / 0.8086 = 5.7605$$

$$R_2 = |4.36602 - 0.773| / 0.8086 = 4.4435$$

Here R_1 and R_2 are larger than its critical value 4.64 and 3.50 respectively. So, 5.43100 and 4.36602 are declared as outliers. From the above calculation it is observed that all the four procedures are able to detect the outliers but not in the same level of significant.

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